Calculate System ID for Ron Beaufort's 'Hot Rod' System with Second Order Lag and Dead Time

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Data :=

	0	1	2	3	4	5	6	7
0	"Date"	"Time"	"Millitm"	"Marker"	0	"Sts_00"	1	"Sts_01"
1	/22/2004"	20:37:51"	471	0	10	0	114	0
2	/22/2004"	20:37:52"	462	0	10	0	114	0
3	/22/2004"	20:37:53"	463	0	10	0	114	0
4	/22/2004"	20:37:54"	465	0	10	0	114	0
5	/22/2004"	20:37:55"	466	0	10	0	114	0
6	/22/2004"	20:37:56"	468	0	10	0	114	0
7	/22/2004"	20:37:57"	469	0	10	0	114	0
8	/22/2004"	20:37:58"	471	0	10	0	114	0
9	/22/2004"	20:37:59"	472	0	10	0	114	0
10	/22/2004"	20:38:00"	474	0	10	0	114	0
11	/22/2004"	20:38:01"	475	0	10	0	114	0
12	/22/2004"	20:38:02"	476	0	10	0	114	0
13	/22/2004"	20:38:03"	478	0	10	0	114	0
14	/22/2004"	20:38:04"	479	0	10	0	114	0
15	/22/2004"	20:38:05"	481	0	10	0	114	0
16	/22/2004"	20:38:06"	482	0	10	0	114	0
17	/22/2004"	20:38:07"	484	0	10	0	114	0

$T := 19$

Sample period in seconds. Adjust T until the ISE is minimized. Ideally this would be automated.

$d := 12$

Dead Time in seconds. It looks like the data above should be about 22 seconds, but it could be there isn't enough resolution in the data. Adjust until ISE is minimized.

$n := 0 .. \dfrac{\text{rows}(\text{Data}) - 2 - d}{T}$

Calculate the number of rows for the PV and CV arrays

$CV_n := \text{Data}_{n \cdot T + 1, 4}$

Copy the data from the Data array into the CV array. Adjust for dead time and for sample time.

$PV_n := \text{Data}_{n \cdot T + 1 + d, 6}$

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Least Squares System Identification

The goal is to calculate the coefficients for Θ for the difference equation below that estimates the temperature. The Φ and y arrays are filled with the PV and CV data for many instances of the equation above. The least squares formula below calculates the Θ coefficients that minimize the errors and provides the best fit.

$n := 0 .. \text{rows}(PV) - 3$

$\Phi_{n,0} := PV_{n+1}$ $\Phi_{n,2} := CV_{n+1}$ $\Phi_{n,4} := 1$

$\Phi_{n,1} := PV_n$ $\Phi_{n,3} := CV_n$ $y_n := PV_{n+2}$

Φ =

	0	1	2	3	4
0	114	114	10	10	1
1	114	114	10	10	1
2	114	114	10	10	1
3	114	114	10	10	1
4	114	114	80	10	1
5	121	114	80	80	1
6	134	121	80	80	1
7	150	134	80	80	1
8	169	150	80	80	1
9	187	169	80	80	1
10	205	187	80	80	1
11	222	205	80	80	1
12	238	222	80	80	1
13	252	238	80	80	1
14	265	252	80	80	1
15	277	265	80	80	1

y =

	0
0	114
1	114
2	114
3	114
4	121
5	134
6	150
7	169
8	187
9	205
10	222
11	238
12	252
13	265
14	277
15	288

$$\Theta := \left(\Phi^T \cdot \Phi\right)^{-1} \cdot \Phi^T \cdot y$$

Formula for least squares estimation.

$$\Theta = \begin{pmatrix} 1.549 \\ -0.586 \\ 0.1 \\ 0.038 \\ 2.786 \end{pmatrix}$$

Θ[0] = A[n-1]
Θ[1] = A[n-2]
Θ[2] = B[n-1]
Θ[3] = B[n-2]
Θ[4] = C

The coefficients for a difference equation for a second order lag system.

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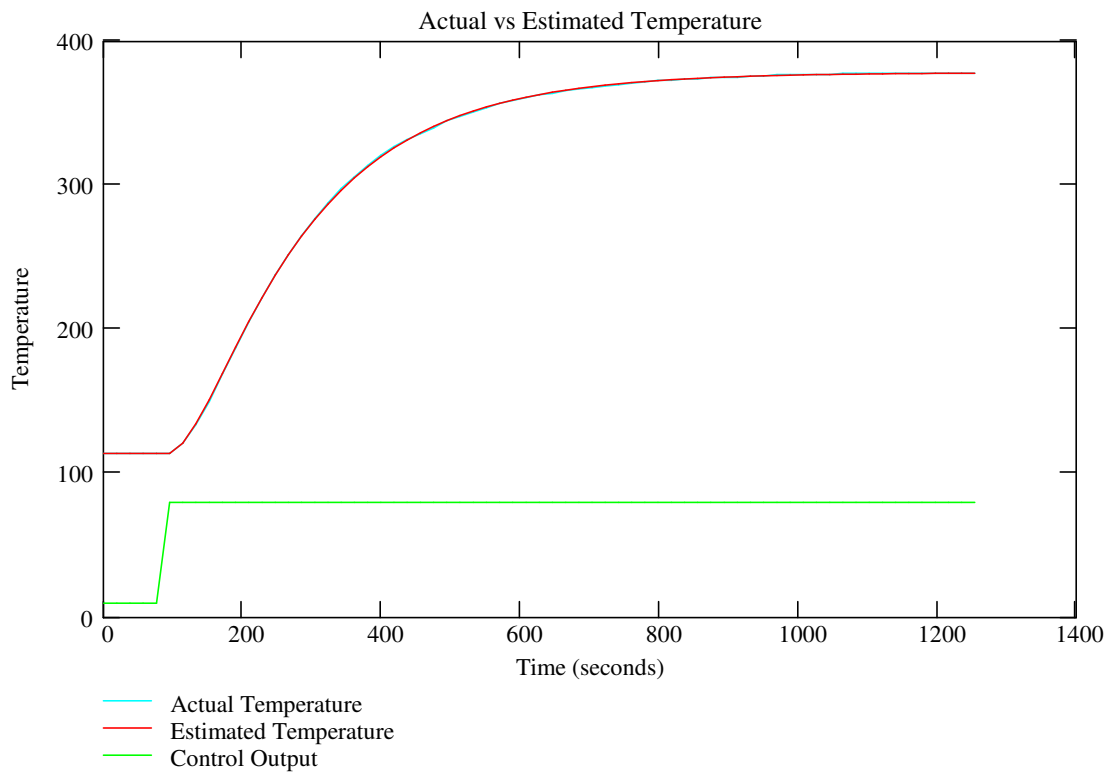## Compare Estimated with Actual Temperature

$n := 0 .. 1$ $\qquad\qquad$ $Est_n := PV_n$

$n := 0 .. rows(PV) - 3$

Initialize the first two estimated values with the actual values as the first two estimation values are not calculated.

$Est_{n+2} := \Theta_0 \cdot Est_{n+1} + \Theta_1 \cdot Est_n + \Theta_2 \cdot CV_{n+1} + \Theta_3 \cdot CV_n + \Theta_4$

The difference equation that estimates the temperature as a function of the control output (CV).

Actual vs Estimated Temperature

Time (seconds)

— Actual Temperature
— Estimated Temperature
— Control Output

$$ISE := \frac{\sum (PV - Est)^2}{rows(PV)}$$ $\qquad$ $ISE = 0.261285$

Verify estimation by comparing actual to estimated using ISE (integrated squared error). The smaller the better.

Calculate System Gains and Time Constants

$$A := \begin{pmatrix} -\Theta_1 \\ -\Theta_0 \\ 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 0.585951 \\ -1.549408 \\ 1 \end{pmatrix}$$

Coefficients of the z domain transfer function from above.

$$zPoles := polyroots(A)$$

$$zPoles = \begin{pmatrix} 0.655475 \\ 0.893933 \end{pmatrix}$$

Poles in the z domain. These should be positive and real

$$sPoles := \frac{\ln(zPoles)}{T}$$

$$sPoles = \begin{pmatrix} -0.022231 \\ -0.005901 \end{pmatrix}$$

Poles in the s domain. These should be negative and real

$$\tau := -\frac{1}{sPoles}$$

$$\tau = \begin{pmatrix} 44.981614 \\ 169.454239 \end{pmatrix}$$

Time constants for model in seconds.

$$G := \frac{\Theta_2 + \Theta_3}{1 - \Theta_0 - \Theta_1}$$

$$G = 3.77666$$

Gain in degrees f per percent output.

$$d = 12$$

Best Dead Time

$$Ta := \frac{\Theta_4}{1 - \Theta_0 - \Theta_1}$$

$$Ta = 76.233399$$

Ambient Temperature

Calculate PID Gain and Time Constants.

$$\tau0 := \frac{\tau_0}{60} \qquad\qquad \tau0 = 0.749694$$

Convert time constants in seconds to time constants in minutes.

$$\tau1 := \frac{\tau_1}{60} \qquad\qquad \tau1 = 2.824237$$

$$\lambda := 1$$

$$3 \cdot \tau0 \cdot \frac{\tau1}{\tau0 + \tau1} = 1.777297$$

Adjust the three real poles using the real time constant λ. Making λ smaller makes the system respond faster. If λ is too small the control will saturate and making the λ smaller will not have any more affect.

Calculate the PID Gains and Time Constants.

$$K := \frac{3 \cdot \tau0 \cdot \tau1 - \lambda^2}{\lambda^2 \cdot G} \qquad\qquad K = 1.417109$$

K is in %output/degree. τi and τd are in minutes.

$$\tau i := \frac{3 \cdot \lambda \cdot \tau0 \cdot \tau1 - \lambda^3}{\tau0 \cdot \tau1} \qquad\qquad \tau i = 2.527703$$

$$\tau d := \lambda \cdot \frac{-3 \cdot \tau0 \cdot \tau1 + \lambda \cdot \tau0 + \lambda \cdot \tau1}{-3 \cdot \tau0 \cdot \tau1 + \lambda^2} \qquad \tau d = 0.519066$$

Calculate Difference Equation Coefficients for PID update rate.

$$\exp\left(\ln(\text{zPoles}) \cdot \frac{1}{T}\right) = \begin{pmatrix} 0.978014 \\ 0.994116 \end{pmatrix}$$
Compute Poles in Z domain with 1 second sampling.

$T := 1$
PID and Simulator Update

$\tau 0 := 60 \cdot \tau 0$ $\qquad$ $\tau 1 := 60 \cdot \tau 1$
Change time constants back to seconds

$a := \dfrac{1}{\tau 0}$ $\qquad\qquad$ $b := \dfrac{1}{\tau 1}$

$Q := \exp(-a \cdot T)$ $\qquad\qquad$ $R := \exp(-b \cdot T)$

$Q = 0.978014$ $\qquad\qquad$ $R = 0.994116$
Should be the sampe as the z poles above.

$A := \dfrac{b \cdot (1 - Q) - a \cdot (1 - R)}{a \cdot b \cdot (b - a)}$ $\qquad$ $B := \dfrac{a \cdot Q \cdot (1 - R) - b \cdot R \cdot (1 - Q)}{a \cdot b \cdot (b - a)}$

$A = 0.495339$ $\qquad\qquad$ $B = 0.490715$

$$\dfrac{A \cdot z + B}{z^2 + (-R - Q) \cdot z + Q \cdot R}$$

$B2 := B \cdot a \cdot b$ $\qquad$ $B2 = 0.000064$ $\qquad$ $B2 \cdot G = 0.000243136$

$B1 := A \cdot a \cdot b$ $\qquad$ $B1 = 0.000065$ $\qquad$ $B1 \cdot G = 0.000245427$

$A2 := Q \cdot R$ $\qquad$ $A2 = 0.972259$

$A1 := -(Q + R)$ $\qquad$ $A1 = -1.97213$

$C := (1 + A1 + A2) \cdot Ta$ $\qquad$ $C = 0.009861852$
Offset for ambient temperature.

Compare Estimated with Actual Temperature

$$N := \frac{\text{rows}(\text{Data}) - 2 - d}{T} \qquad n := 0 .. N$$

Calculate the number of rows for the PV and CV arrays

$$CV_n := \text{Data}_{n \cdot T + 1, 4}$$

Copy the data from the Data array into the CV array. Adjust for dead time and for sample time.

$$PV_n := \text{Data}_{n \cdot T + 1, 6}$$
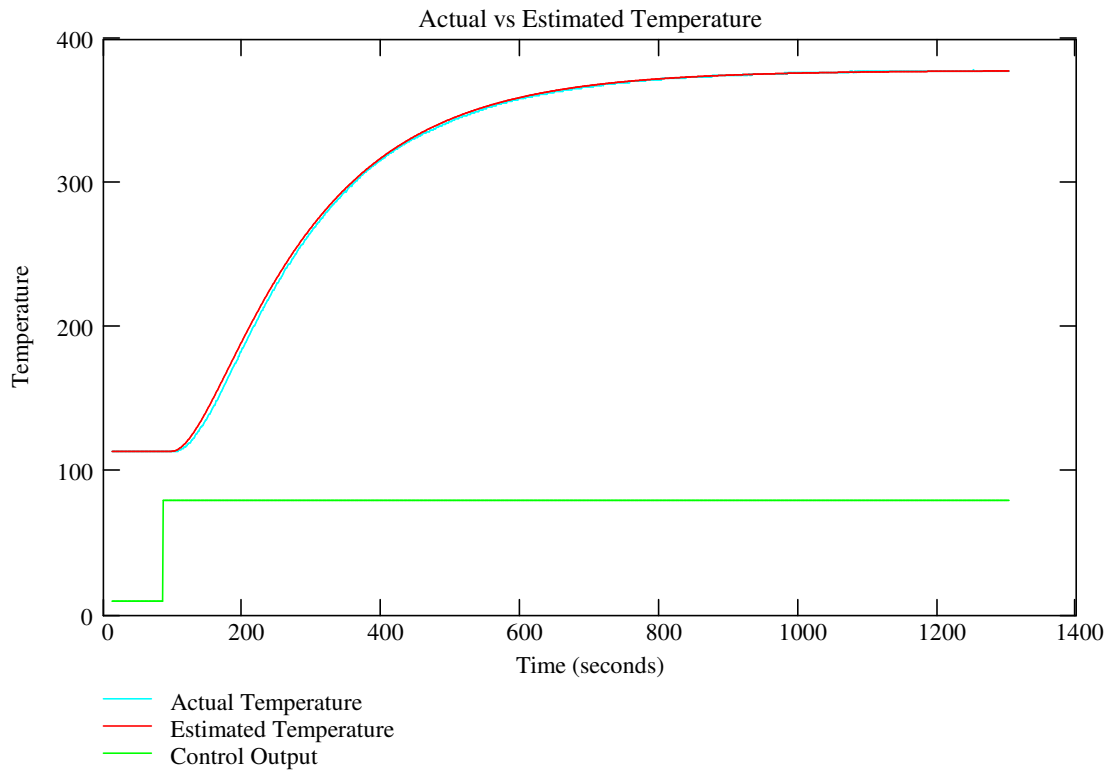
$$n := 0 .. d + 2 \qquad Est_n := PV_n$$

Initialize the first two estimated values with the actual values as the first two estimation values are not calculated.

$$n := d .. \text{rows}(PV) - 3$$

$$Est_{n+2} := -A1 \cdot Est_{n+1} + -A2 \cdot Est_n + G \cdot B1 \cdot CV_{n+1-d} + G \cdot B2 \cdot CV_{n-d} + C$$

The difference equation that estimates the temperature as a function of the control output (CV).

Notice the indices for CV are delayed by the dead time.

Actual vs Estimated Temperature

— Actual Temperature
— Estimated Temperature
— Control Output

$$ISE := \frac{\sum (PV - Est)^2}{\text{rows}(PV)} \qquad ISE = 4.124598$$

Verify estimation by comparing actual to estimated using ISE (integrated squared error). The smaller the better.

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For RS500 PID

The calculated gain is in %output per degree error. This must be converted to a unitless gain for the RS500 PID. This means the calculated gain must be scaled by the input and output scales.   The input scale is 500 degress/16384 counts and the output scale is 16384 counts per 100% output.

$K = 1.417109$ %Output per degree error

$$Kc := K \cdot \frac{500}{16384} \cdot \frac{16384}{100} \qquad Kc = 7.085543 \qquad \text{Unitless}$$